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Technical Notes

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Effect of Thermal Creep on Buoyancy-Induced Flows in Low-Gravity

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Introduction

IT has been recognized for some time, that a temperature gradient tangential to a solid surface, will induce a slip flow of gas over the surface in the direction of the gradient. This phenomenon, known as "thermal creep," is well known to aerosol scientists as the driving force behind the small particle transport mechanisms of thermophoresis and photophoresis. When length scales larger than the few micrometers associated with aerosol particles are considered—such as boundary-layer flow over an unevenly heated flat plate—thermal creep is invariably neglected in comparison to the buoyancy-induced flow. However, the current interest in microgravity appplications [i.e., chemical vapor deposition (CVD), thin film, and crystal growth] creates situations in which the role of thermal creep in relatively large-scale systems requires reassessment.

Thermal creep can be explained from simple kinetic theory arguments. Molecules striking the surface, that originate from the hotter regions of the gas, impart more momentum to the surface than molecules originating from the cooler regions. To compensate for the uneven momentum transfer, the surface "pushes" against the gas, creating a slip flow over the surface directed towards the hotter gas. From a continuum flow regime viewpoint, the creep velocity can be phenomenologically related to the wall tangential temperature gradient by[1,2]

$$u_c = \frac{c_s \mu R}{P} \frac{\partial T_w}{\partial x} \tag{1}$$

where c_s is the coefficient of thermal slip (≈ 1.1 for diffusely-reflecting surfaces[2]), and μ, R, and P are the dynamic viscosity, gas constant, and pressure of the gas, respectively. It should be emphasized that thermal creep, unlike the often-associated phenomena of temperature-jump and viscous-slip, is not confined to rarefied flow regimes or length scales on the order of the gas mean-free-path. All the quantities appearing in Eq. (1) are continuum properties of the gas.

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Equation (1) reveals that the creep velocity does indeed "creep" along—in that a considerable temperature gradient is required to set up a moderate velocity (e.g., 10^5 K/m for $u_c \approx 1$ cm/s at STP). However, Rosner[3] recently reported that the temperature gradients and low pressures associated with certain microgravity CVD ampoules are sufficient to lead to significant creep-induced bulk flows. Such conclusions are important when considering that the ostensible purpose of microgravity is to suppress bulk flow through the elimination of buoyancy. Thermal creep would also be significant in the vicinity of a flame anchored to a solid surface, and could potentially affect the flame-spread rate under microgravity conditions.

The objective of this work is to examine the the relative importance of thermal creep flow in the presence of buoyancy-induced flow, using a well-studied geometrical configuration of flow past a vertical flat plate with a nonuniform surface temperature distribution. To provide an estimation of the key quantities of interest (i.e., heat transfer and entrained flow) from a relatively simple analysis, we model the problem using a constant-property boundary-layer formulation. The accuracy of this approach is certainly questionable. In the limit of negligible buoyancy, a thermal creep velocity of sufficient magnitude to validate the boundary layer assumptions, would require a large surface tangential temperature gradient—resulting in significant variation in fluid density in the streamwise direction. In spite of these limitations, the analysis presented here should provide a qualitative description of the role of thermal creep on the transport properties of fluid under microgravity conditions.

Analysis

Consider a vertical flat plate oriented parallel to the direction of the gravitational acceleration and occupying the positive half of the plane $y = 0$. Following Sparrow and Gregg,[4] we seek similarity solutions to the problem when the surface temperature of the plate varies as $T_w = T_\infty + Nx^n$, where T_∞ is the temperature of the quiescent ambient fluid far away from the plate, and N is a positive constant. Assuming steady, laminar flow, and constant properties, the boundary-layer equations expressing mass, momentum, and energy conservation are written

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{4}$$

with the boundary conditions

$$\left.\begin{aligned} u &= u_c(x) \\ v &= 0 \\ T &= T_w(x) \end{aligned}\right\} y = 0, \qquad \left.\begin{aligned} u &\to 0 \\ T &\to T_\infty \end{aligned}\right\} y \to \infty \tag{5}$$

In the above, ν and α are the momentum and thermal diffusivity of the gas, g is the gravitational acceleration, β is the coefficient of thermal expansion, and $u_c(x)$ is the thermal creep flow at the surface given by Eq. (1). In writing the boundary conditions, it has been assumed that the gas mean-free-path is significantly less than the boundary-layer thickness. Therefore, temperature jump and visous slip effects are neglected.

By defining appropriate dimensionless similarity variables η, f, and θ, the governing differential equations can be reduced in the usual manner to a system of nonlinear ordinary differential equations. The choice of a characteristic velocity leading to a similarity, is actually not unique, because the problem contains two clearly definable velocity scales—creep and buoyant—and either one of them could be chosen. Sparrow et al.[5] encountered a similar situation in their formulation of combined forced and free convection over a vertical flat plate. Since we are interested in the heat and mass transfer characteristics in the limit of vanishing gravitational acceleration, it is appropriate to select creep velocity as the characteristic velocity scale, and define η using u_c as

$$\eta = (y/x)\sqrt{(Re_x/2)} \tag{6}$$

where

$$Re_x = (u_c x/\nu) = (nc_s N x^n/T_\infty) \tag{7}$$

The dependent variables are defined as

$$u = (\nu/x)Re_x f' \tag{8}$$

$$v = -(\nu/x)\sqrt{(Re_x/2)}[nf + (n-2)\eta f'] \tag{9}$$

$$(T - T_\infty) = (T_w - T_\infty)\theta(\eta) \tag{10}$$

It can easily be shown that the problem admits similarity only for the case of $n = 3$, i.e., $T_w = T_\infty + Nx^3$. The governing equations then reduce to

$$f''' + 3f''f - 4(f')^2 + 2(Gr/Re^2)\theta = 0 \tag{11}$$

$$\theta'' + Pr[3f\theta' - 6f'\theta] = 0 \tag{12}$$

with the boundary conditions

$$\left.\begin{matrix} f = 0 \\ f' = 1 \\ \theta = 1 \end{matrix}\right\} \eta = 0, \qquad \left.\begin{matrix} f' \to 0 \\ \theta \to 0 \end{matrix}\right\} \eta \to \infty \tag{13}$$

The parameters in the above equations are the Prandtl number Pr and Gr/Re^2, where the Grashof number Gr is defined

$$Gr = (g\beta N x^6/\nu^2) \tag{14}$$

The system of Eqs. (11) and (12), along with the boundary conditions in Eq. (13), form a well-posed two-point boundary value problem. The problem is solved numerically using the software package COLSYS[6] for a range of values for the parameter Gr/Re^2, and a fixed value of the Prandtl number Pr of 0.7. The results of the calculations are presented in the next section.

Results

It is of interest to examine the dependence of Gr/Re^2 upon the physically-controllable quantities N, g, and P. Denoting Nx^3 as ΔT_w, one finds that

$$(Gr/Re^2) = (gT_\infty x^3/9c_s^2 \Delta T_w \nu^2) \tag{15}$$

For a situation in which the gas is air at 1 atm pressure, and the temperature varies from 300 to 800 K over a distance of 10 cm, Gr/Re^2 will be on the order of 10^5–10^6 at normal gravity. Consequently, for small g conditions it is appropriate to examine the solution for Gr/Re^2 around 1–1000.

Presented in Fig. 1 is a plot of the dimensionless velocity f' vs η for Gr/Re^2 ranging between 0–100 and $Pr = 0.7$. In the limit $Gr/Re^2 \to \infty$, corresponding to either relatively large g and/or P, or small ΔT_w, one recovers the purely natural convective solution obtained by Sparrow and Gregg.[4] As Gr/Re^2 decreases, two effects become apparent, namely 1) the boundary layer thickness increases; and 2) the shear stress at the wall [reflected in $f''(0)$] goes from negative to positive values. The reversal in $f''(0)$ occurs for $Gr/Re^3 = 4.3305$, and for smaller values of Gr/Re^2 the location of maximum velocity within the boundary layer occurs directly at the wall.

With regard to microgravity conditions, perhaps the most important quantity arising from thermal creep, is the total amount of fluid entrained into the boundary layer from the bulk fluid and converted downstream along the plate. Denoting the entrainment mass flow rate per unit width as $\dot{m}$, it follows that

$$\dot{m} = \rho \int_0^\infty u \, dy \tag{16}$$

which, in terms of the nondimensional variables, can be expressed

$$(\dot{m}/\rho\nu) = \sqrt{2Re} f(\infty) \tag{17}$$

In the limit of vanishing buoyancy ($Gr/Re^2 \to 0$), and dominant buoyancy ($Gr/Re^2 \to \infty$), $f(\infty)$ displays the limiting behavior

$$f(\infty) = 0.5570, \qquad (Gr/Re^2) = 0 \tag{18}$$

$$f(\infty) \to 0.5603(Gr/Re^2)^{1/4}, \qquad (Gr/Re^2) \to \infty \tag{19}$$

Another quantity of interest is the Nusselt number $Nu = hx/k$, where h is the heat transfer coefficient and k is the gas thermal conductivity. Here, a result identical to the forced convection relation[5] is obtained

$$Nu = -(Re/2)^{1/2}\theta'(0) \tag{20}$$

In the pure creep and pure buoyancy flow limits, one obtains

$$\theta'(0) = -1.822, \qquad (Gr/Re^2) = 0 \tag{21}$$

$$\theta'(0) \to -0.9674(Gr/Re^2)^{1/4}, \qquad (Gr/Re^2) \to \infty \tag{22}$$

Presented in Fig. 2 are plots of $f(\infty)$ and $\theta'(0)$ vs $Gr^{1/2}/Re$. Thermal creep effects become significant for $Gr^{1/2}/Re$ less than approximately 20, and become dominant compared to buoy-

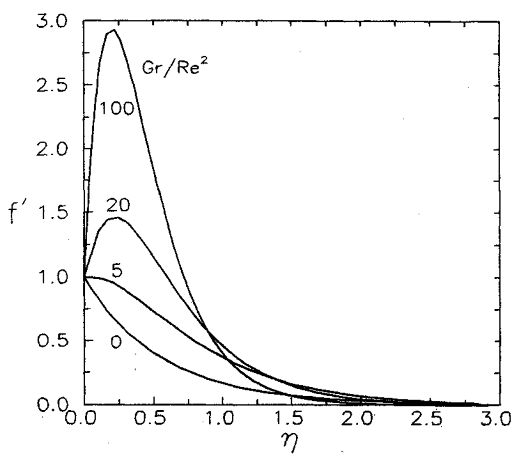

Fig. 1 Dimensionless velocity f'.

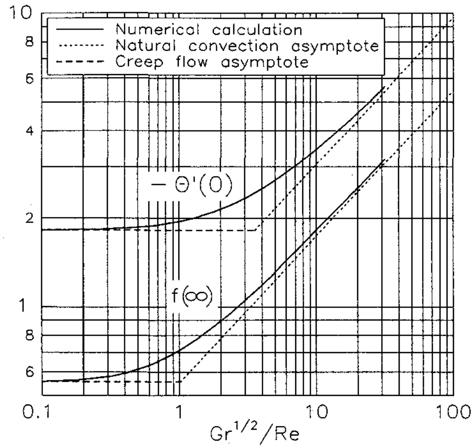

Fig. 2 Dimensionless heat transfer $\theta'(0)$ and entrained flow $f(\infty)$ rates.

Table 1 Numerical results

Gr/Re^2	$f(\infty)$	$f''(0)$	$-\theta'(0)$
0	0.5570	−1.907	1.823
0.1	0.5847	−1.850	1.841
1	0.7165	−1.396	1.949
4.3305	0.9146	0	2.160
10	1.082	2.015	2.369
100	1.819	24.02	3.449
1000	3.175	154.4	5.665

ancy for $Gr^{1/2}/Re$ less than 1. To illustrate the creep effects in dimensional terms, for the parameters $\Delta T_w = 500$ K and $x = 0.1$ m, a value of $g = 10^{-5}\ g_0$ corresponds to $Gr^{1/2}/Re \approx 5$ at one atmosphere pressure. For these conditions, $\dot{m}/\rho\nu$ and Nu, computed with consideration of thermal creep, are approximately 6 and 25% greater, respectively, than that computed by a purely free convection analysis. At lower pressures, the effects of thermal creep will be even more apparent, because $Gr^{1/2}/Re$ will (through the dependence of ν) be proportional to pressure.

Numerical values of the quantities $f(\infty)$, $f''(0)$ and $-\theta'(0)$ are listed in Table 1 as a function of Gr/Re^2.

Conclusions

The intention of this work has been to provide an estimation of the effect of thermal creep on laminar, natural convection boundary layers. We find that thermal creep can significantly alter flow and heat transfer characteristics in situations where buoyancy becomes negligible. The results have important implications on the development and design of applications that are intended to exploit buoyancy-free microgravity environments, i.e., CVD and crystal growth processes.

Acknowledgments

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References

[1]Kennard, E. H., *Kinetic Theory of Gases,* McGraw-Hill, New York, 1938, pp. 327–330.

[2]Talbot, L., Cheng, R. K., Schefer, R. W., and Willis, D. R., "Thermophoresis of Particles in a Heated Boundary Layer," *Journal of Fluid Mechanics,* Vol. 101, Pt. 4, 1980, pp. 737–758.

[3]Rosner, D. E., "Side Wall Gas 'Creep' and 'Thermal Stress Convection' in Microgravity Experiments on Film Growth by Vapor Transport," *Physics of Fluids,* Vol. A1, No. 11, 1989, pp. 1761–1763.

[4]Sparrow, E. M., and Gregg, J. L., "Similar Solutions for Free Convection from a Non-Isothermal Vertical Plate," *Transactions of the American Society of Mechanical Engineers,* Vol. 80, 1958, pp. 379–386.

[5]Sparrow, E. M., and Eichhorn, R., and Gregg, J. L., "Combined Forced and Free Convection in a Boundary Layer Flow," *Physics of Fluids,* Vol. 2, 1959, pp. 319–328.

[6]Ascher, U., Christiansen, J., and Russell, R. D., "Collocation Software for Boundary-Value ODEs," *Association for Computing Machinery Trans. Math. Software,* Vol. 7, 1981, pp. 209–222.